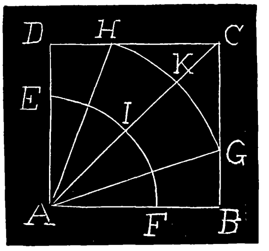
Since no law of distribution of the events is given, we will assume that the intersections of the arcs with the line AC are uniformly distributed on AC.

Hence, if the random arc intersects AC between I and K its length will exceed a, the length of a side of the square.



Hence, the required probability is

$$p=\frac{IK}{AC}=\frac{AK-AI}{AC}.$$

But, since $EIF=\frac{1}{2}\pi.AI=a$, $AI=2a/\pi$, and likewise, $AK=a/\theta$, where $\theta=\angle HAG$.

$\therefore p=\frac{(a/\theta-2a/\pi)}{a\sqrt{2}}=\frac{1}{\theta\sqrt{2}}-\frac{\sqrt{2}}{\pi}$, where θ may be found by the method of Double Position from the equation, $\cos(\frac{1}{4}\pi-\frac{1}{2}\theta)=\theta$.

For $\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)=\frac{AB}{AG}=\frac{a}{AG}$, and $\theta.AG=HKG=a$.

Hence, $\theta=\frac{a}{AG}$, and therefore $\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)=\theta$.

Solving this equation we find $\theta=.95266$ radians.

$$\therefore p=\frac{1}{.95266\sqrt{2}}-\frac{\sqrt{2}}{\pi}=.2920+.$$

Also solved by F. P. Matz who distributes the events in proportion to the area between the limiting arcs and the area of the square; G. B. M. Zerr, who gets as a result by using the calculus, .0734; and Henry Heaton, who finds the mean lengths of the arcs.

159. Proposed by J. E. SANDERS, Hackney, Ohio.

A box contains n tickets numbered from 1 to n. How many diaws, on the average, will it take to draw all the numbers, each ticket being replaced before drawing again? What is the numerical result for $n=2$ and $n=6$?

Solution by W. W. LANDIS, A. M., Professor of Mathematics, Dickinson College, Carlisle, Pa.

The chance of drawing any particular number at least once in p drawings is

$$1-\left(1-\frac{1}{n}\right)^p.$$

The chance that all will be drawn in p drawings (p being, of course $>n$) is

$$\left[1-\left(1-\frac{1}{n}\right)^p\right]^n,$$

which by the conditions of the problem must equal $\frac{1}{2}$. Solving this equation for p,

$$p=\frac{\frac{1}{n}\log 2-\log(2^{1/n}-1)}{\log n-\log(n-1)}.$$

For $n=2$, $p=1.75+$, hence two drawings must be made; for $n=6$, $p=12.15+$; hence, thirteen drawings must be made.

Also solved with the same result by F. O. Whitlock, and by a different method, which seems to me to be incorrect, by S. A. Corey, Hiteman, Iowa. The problem is equivalent to that of Problem IX, page 52, of Meyer's *Wahrscheinlichkeitsrechnung*. F.

MISCELLANEOUS.

146. Proposed by F. P. MATZ, Ph. D., Sc. D., Reading, Pa.

The year 1905 *began*, and will *end*, on a Sunday. Prove that this can not occur again until the year 2015.

Solution by WILLIAM HOOVER, Ph. D., Athens, Ohio.

The Dominical Letter for Sunday when on January 1 is A, and also when on December 31, the year being common. Those common years in the present century fulfilling the required conditions must have A for their Dominical Letter; such years are 1905, 1911, 1922, 1933, 1939, 1950, 1961, 1967, 1978, 1989, 1995, sufficient to show that the statement in the problem is not true.

REMARK BY PROPOSER. The year 2015 will begin on a Thursday.
Also solved by A. H. Holmes, Henry Heaton, G. B. M. Zerr, and G. W. Greenwood.

147. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

If an *unknown* curve be described under a constant acceleration not tending to the center and the hodograph is a cardioid, what is the unknown curve?

I. Solution by WILLIAM HOOVER, Ph. D., Athens, Ohio.

Let r and p be the radius vector and perpendicular upon the tangent to the curve at the outer extremity of r and r'; p' the analogous lines in the hodograph, and h the double area generated by r in a unit of time.

Then by the theory of the hodograph,

$$r'=\frac{h}{p}\,\ldots\ldots\,(1),\quad p'=\frac{h}{r}\,\ldots\ldots\,(2).$$

Also, from the theory of central forces,

$$k=\frac{h^2}{p^3}\frac{dp}{dr}\,\ldots\ldots\,(3),$$

and for the cardioid,

$$p'^2=\frac{r'^3}{2a}\,\ldots\ldots\,(4).$$